

Estimation of Kinetic Friction Coefficient for Sliding Rigid Block Nonstructural Components

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Summary

The kinetic friction coefficient between a sliding rigid block and its supporting floor is estimated. The block can be used to model a broad range of nonstructural components such as free standing and unrestrained building contents, and mechanical and electrical equipments. Experimental results and theoretical considerations are used to estimate this coefficient, since the coefficient of kinetic friction cannot be measured directly. Estimates of kinetic friction coefficient are obtained using acceleration and displacement-based methods for carpet on steel interface. These estimates account implicitly for the uncertainty in experimental errors, imperfections in block-floor interfaces, and the relationship between the kinetic friction coefficient and the loading, and the block size. It is shown that most of the pairs of acceleration and displacement-based estimates of the kinetic coefficient of friction are included in the 90% probability contour of these parameters assumed to be Gaussian.

Introduction

In recent studies it has been shown that, for a facility, the financial consequences of seismic occurrences result mainly from the poor performance of its nonstructural components and systems (Gould and Griffin 2003; Kircher 2003). The seismic performance of free standing and unrestrained components, for example, mechanical and electrical equipments, depends strongly on the kinetic friction coefficient between the components and the supporting floors. However, the kinetic friction coefficients for common component-floor interfaces are not available in the literature (Soler and Singh 1984; Garcia and Soong 2003; Castiglioni 2003).

The objective of this study is to estimate the kinetic friction coefficient between a sliding rigid block and its supporting floor. The block can be used to model a broad range of nonstructural components such as mechanical and electrical equipments, free standing and unrestrained building contents. Since the coefficient of kinetic friction cannot be measured directly, experimental results and theoretical considerations are needed to estimate this coefficient. Estimates of kinetic friction coefficient are obtained for carpet on steel interface, using acceleration and displacement-based methods. It is shown that most of the pairs of acceleration and displacement-based estimates of the kinetic coefficient of friction are included in the 90% probability contour of these parameters assumed to be Gaussian random variables with the second moment properties estimated from experiments.

Dynamic Analysis of Block-Table System

Consider a free standing rigid block of mass m , which can represent a sliding nonstructural component, sitting on a shake table. Let $z(t)$ and $y(t)$ be the displacement of the shake table and of the block relative to an absolute frame whose origin corresponds to the vertical position of the block centroid, respectively, μ be the coefficient of kinetic friction between the block and the surface of the shake table, and g be the acceleration of the gravity as shown in Figure 1. Denote the displacement of the block relative to the shake table by $x(t)$, so that $y(t)=z(t)+x(t)$.

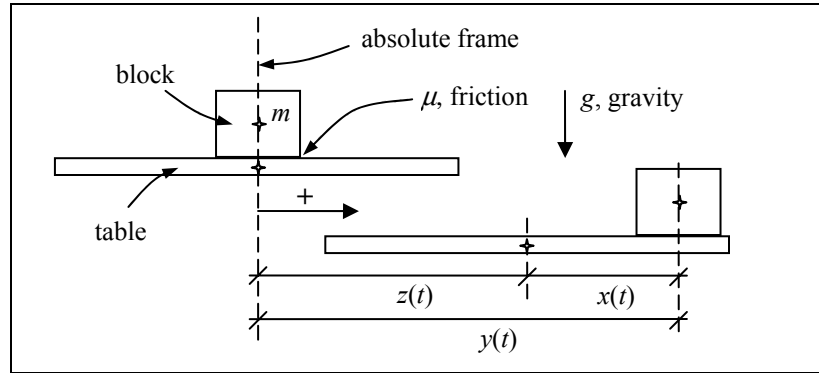


Figure 1. System of sliding block and shaking table

The equation of motion for the block is

$$\ddot{x}(t) = \begin{cases} -\ddot{z}(t) - \mu g \operatorname{sign}(\dot{x}(t)), & \text{sliding condition} \\ 0, & \text{sticking condition} \end{cases} \quad (1)$$

where $\operatorname{sign}(b)=-1, 0, 1$, for $b<0, b=0$ and $b>0$, respectively. The shake table acceleration used in the experiments performed at the University at Buffalo has the form

$$\ddot{z}(t) = w(t)\alpha g \sin(\nu t), \quad t \in [0, t_f] \quad (2)$$

where $w(t)$ is a modulation function increasing to 1 and starting at $w(0)=0$.

A computer algorithm for calculating the block relative acceleration, velocity and displacement is presented in Kafali and Grigoriu (2005). Figure 2 shows the calculated total and relative acceleration and velocity responses of the block, and the acceleration and the velocity of the shake table for $\mu=0.2, \alpha=0.8, \nu=2\pi T, T=0.5$ sec and $t_f=3.1 T$, in Eqs. 1 and 2.

The calculated steady state total block acceleration, velocity and displacement responses have zero temporal mean, and are symmetric about the time axis, under the sinusoidal excitation defined in Eq. 2 because of the symmetry of the equation of motion. On the other hand, recorded block responses exhibit drifts since the properties of the block-table interface exhibit spatial variation. To relate calculated results to experimental results, block response records need to be corrected, as shown in the following sections.

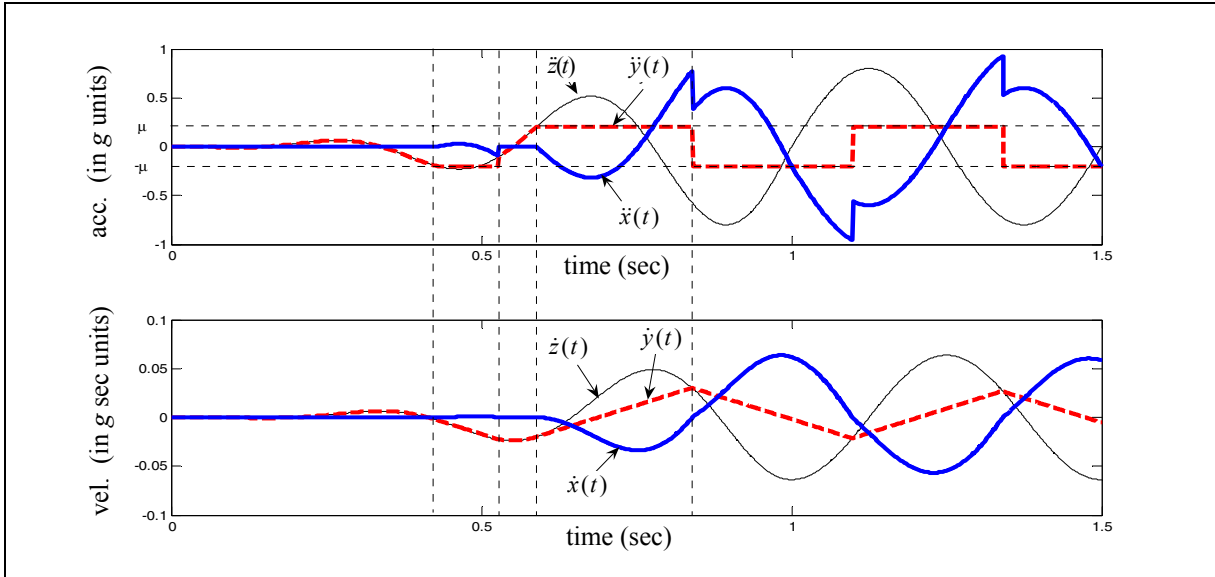


Figure 2. Algorithm

Estimation of Kinetic Friction Coefficient

The coefficient of kinetic friction μ is estimated using the maximum responses of the block obtained through experiments at the University at Buffalo, and a relationship between the maximum responses and the kinetic friction coefficient, obtained at Cornell University using theoretical considerations. The maximum absolute total acceleration response, $\max_t |\ddot{y}(t)|$, of a block subjected to the input acceleration defined by Eq. 2, is obtained using Eq. 1

$$\max_t |\ddot{y}(t)| = \begin{cases} \mu g, & \text{sliding condition} \\ \max_t |\ddot{z}(t)| = \alpha g, & \text{sticking condition} \end{cases} \quad (3)$$

It is difficult to obtain a relationship in closed form between the maximum absolute displacement response $\max_t |y(t)|$ and the kinetic friction coefficient μ . The algorithm defined in Kafali and Grigoriu (2005) is used to relate $\max_t |y(t)|$ and μ . Figure 3 shows an example of maximum total displacement versus kinetic coefficient friction curve for $\mu \in [0.1, 0.9]$, $\alpha = 0.8$, $\nu = 2\pi T$, $T = 0.5$ sec, $t_f = 40.2 T$.

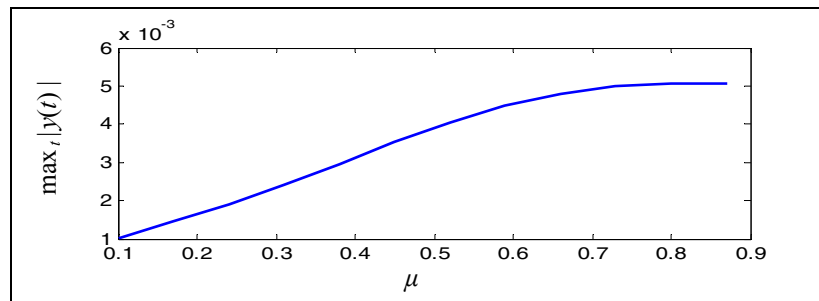


Figure 3. Relation between $\max_t |y(t)|$ and μ

Data Analysis

A series of shake table experiments on rigid blocks have been performed at the University at Buffalo to characterize the kinetic friction coefficient carpet on steel interface (explained in detail in Fathali 2005). The surface of the shake table is steel and the carpet is attached to the bottom of the blocks. The excitations are modulated unidirectional sine waves with different amplitudes and frequencies to account for the uncertainty related to the dependence of the kinetic friction coefficient on the loading. To account for the uncertainty related to the experimental errors, six nominally identical blocks are tested simultaneously for given amplitude, period pair. Two methods are used to estimate the kinetic coefficient of friction. The methods are based on acceleration and displacement records of blocks obtained in Fathali (2005).

Acceleration-Based Estimates of Kinetic Friction Coefficient: Let $a(t)$, $t \in [0, t_f]$, be the recorded acceleration time history of a block in a given test, with respect to a fixed frame. The corresponding block acceleration in calculations is denoted by $\ddot{y}(t)$. The following 4-step procedure was used to find estimates μ_{acc} of μ based on displacement records (details can be found in Kafali and Grigoriu 2005).

- Step-1: The acceleration record $a(t)$ is corrected by subtracting its temporal mean.
- Step-2: The steady state part of the corrected acceleration response record $a_{ss,c}(t)$ is obtained.
- Step-3: The maximum absolute acceleration $\max_t |a_{ss,c}(t)|$, is estimated by its most likely value from the histogram of $|a_{ss,c}(t)|$.
- Step-4: The coefficient of kinetic friction is obtained using Eq. 3.

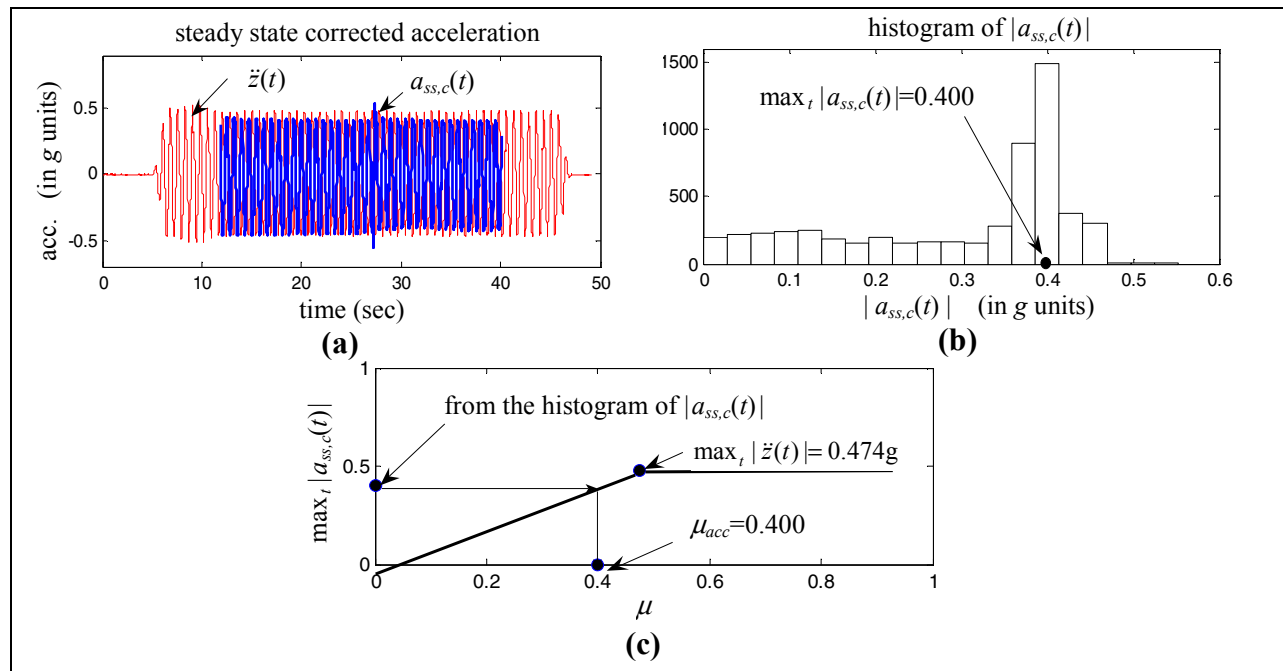


Figure 4. Acceleration-based estimation (test 65, block 1)

Displacement-Based Estimates of Kinetic Friction Coefficient: Let $d(t)$, $t \in [0, t_f]$, be the recorded displacement time history of a block in a given test, with respect to a fixed frame. The corresponding block displacement in calculations is denoted by $y(t)$. The following 4-step procedure was used to find estimates μ_{disp} of μ based on displacement records (details can be found in Kafali and Grigoriu 2005).

- Step-1: The displacement record $d(t)$ is corrected by subtracting its drift.
- Step-2: The steady state part of the corrected displacement response record $d_{ss,c}(t)$ is obtained.
- Step-3: The maximum absolute acceleration $\max_t |d_{ss,c}(t)|$, is estimated by its most likely value from the histogram of $|d_{ss,c}(t)|$.
- Step-4: The kinetic friction coefficient is obtained using the relation between $\max_t |d_{ss,c}(t)|$ and μ .

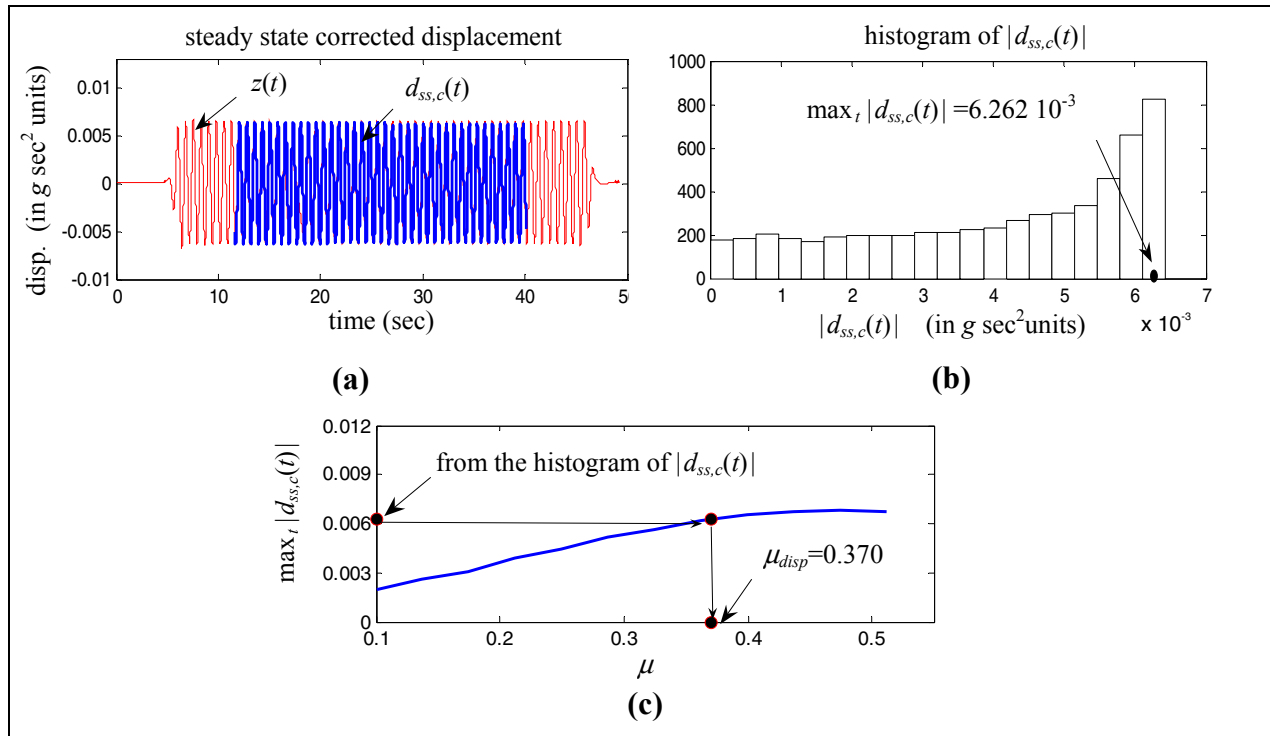


Figure 5. Displacement-based estimation (test 65, block 1)

Results

For a given interface type, the coefficient of kinetic frictions are obtained using the acceleration and displacement responses of all the blocks in each test for that interface type following the procedures described above. Figure 4 shows the (μ_{acc}, μ_{disp}) pairs obtained for carpet-steel interface. Statistics of μ_{acc} and μ_{disp} are calculated using (i) only the small blocks, (ii) only the large blocks, and (iii) all the blocks, are shown in Table 1 for carpet-steel interface.

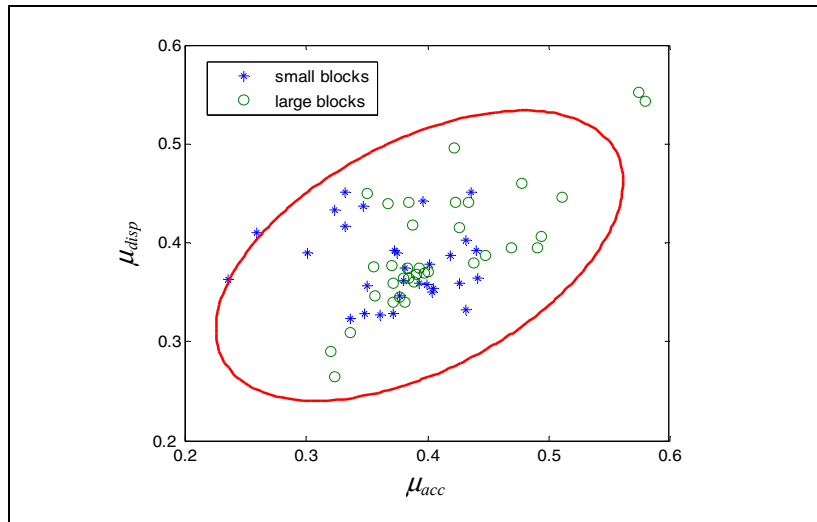


Figure 6. Friction coefficients

It is observed that the correlation between μ_{acc} and μ_{disp} obtained using the large blocks is significantly higher than the correlation obtained using the small blocks. This suggests that the estimates of μ based on small blocks have more noise than those corresponding to large blocks. However, the estimated means and the coefficients of variation are insensitive to block size. It is concluded that there is no apparent size effect in the estimates of μ_{acc} and μ_{disp} .

Table 1. Statistics of kinetic friction coefficient

| | Small blocks | | Large blocks | | All blocks | |
|---------------------------------|--------------|--------------|--------------|--------------|-------------|--------------|
| | μ_{acc} | μ_{disp} | μ_{acc} | μ_{disp} | μ_{acc} | μ_{disp} |
| Mean | 0.374 | 0.379 | 0.410 | 0.394 | 0.393 | 0.387 |
| Standard Deviation | 0.051 | 0.038 | 0.063 | 0.062 | 0.060 | 0.053 |
| Coefficient of variation | 0.137 | 0.101 | 0.154 | 0.157 | 0.153 | 0.136 |
| Correlation coefficient | -0.100 | | 0.752 | | 0.509 | |

Assuming that μ_{acc} and μ_{disp} are correlated Gaussian random variables with means, standard deviations and correlation coefficients given in Table 1, contour lines of the joint probability density function of μ_{acc} and μ_{disp} corresponding to 90% probability are shown in Figure 4. Most of the data points are in the 90% probability contour.

Concluding Remarks

The kinetic friction coefficient between a sliding rigid block and its supporting floor for carpet on steel interface is estimated using experimental results and theoretical considerations. Estimates of the kinetic friction coefficient are obtained using acceleration and displacement-based methods, and

account implicitly for the uncertainty in experimental errors, imperfections in block-floor interfaces, and the relationship between the kinetic friction coefficient and the loading, and the block size. It is shown that most of the pairs of acceleration and displacement-based estimates of the kinetic coefficient of friction are included in the 90% probability contour of these parameters assumed to be Gaussian random variables with the second moment properties estimated from experiments.

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