Study of Rotational Column with Plastic Hinge

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Overview:

- Discuss FAST MOST experiment and function of plastic hinge
- Describe test setup
- Discuss analytical model and design considerations
- Present experimental results
- Conclude on behavior and design of plastic hinge
Section I: Background and Test Setup
Background Information:

- Column setup was built for the FAST-MOST hybrid testing experiment
- One of five scaled down columns modeling supports for a bridge
- Goal is to replicate a complete bridge in real time through networking with other test sites
Test Setup:

- Column is subjected to lateral displacement at the top in quasi-static loading and is fixed at the base.
- Top forces and deflections sent as input to other test sites.
- Simulates earthquake-induced ground motion, including pseudo-dynamic response of bridge.

The column-hinge system, with scale.
Our Focus: Plastic Hinge

- Pinned connection between column and foundation
- Rotation prevented by small vertical members
- These small bars act as fuses, allowing rotation before yielding of column
  - Called “Fuse Bars”
Function of Plastic Hinge:

- Type of energy dampening device
- Yielding and subsequent deflection of fuse bars absorb energy
- Reduces fatigue in column during repeated ground motion
Function of Plastic Hinge:

- Post-earthquake, fuse bars can be replaced relatively easily
- Column-hinge system is then restored to original condition
Section II:
Analytical Model and Design Considerations
Research Proposal:

- Develop an accurate model of column-hinge system in the elastic range of the fuse bars
- Consider two design conditions for plastic hinge
  1. Hinge must become plastic before column yields
  2. Limit deflection at top of column for stability of the overlying structure
- Plastic behavior of fuse bars not tested
Model of Hinge: Effective Length of Fuse Bars

- Length able to bend is portion between inside nuts ($l = 2 \frac{1}{4}'\)"
- Axially deformable length depends on direction of elongation
  - Length in tension (blue)
  - Length in compression (red)
  - Take effective area of threads into account
  - Average is the effective length for axial elongation ($l_e = 3.46''$)
Model for Hinge Motion

- Moment about pin will cause rotation of hinge
- Resisted by internal forces of fuse bars

\[ \Delta = \theta r \]

**Rigid Plate**

\[ k = \frac{EA}{le} \]

**Internal Forces in Fuse bar**

\[ F_{axial} = \frac{EA}{le} \theta r \]

\[ M_{bending} = \frac{EI}{l} \theta \]
Buckling Analysis

Consider the two extreme cases of Buckling

<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>$P_{cr}$ (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantilever</td>
<td>2.0</td>
<td>17.4</td>
</tr>
<tr>
<td>Fixed-Fixed</td>
<td>1.0</td>
<td>69.5</td>
</tr>
</tbody>
</table>

$P_{cr}$ is the buckling load

$L_{E}$ is the Euler length

At a load of 17.4 kips (lower extreme), axial stress in the fuse will equal 157.2 ksi.

Fuse-bars yield at approx. 100 ksi

Buckling will not occur.
Angular Stiffness of Hinge

- Moment induced about pin by external load
- Resisting moment generated by axial force and bending moment

\[ \Sigma M_{pin} = 0 \]
\[ M_{hinge} - 4(M_{axial} + M_{bending}) = 0 \]

\[ M_{bending} = \frac{EI}{l} \theta \]
\[ M_{axial} = \frac{EA r^2}{l_e} \theta \]

Angular Stiffness

\[ K_{\theta} = \frac{M_{hinge}}{\theta} = 4 \left( \frac{E A r^2}{l_e} + \frac{E I}{l} \right) \]
Flexural Stiffness of Column

\[ K_{\text{flex}} = \frac{P}{\delta_{\text{flex}}} \]

- Properties of S3x5.7 column taken from ASCI manual
- Additional flanges and connecting plates assumed to be infinitely stiff

Flexural Flexibility Equation

\[ \delta_{\text{flex}} = \frac{PL_c^3}{EI} \left[ 1 - \left( \frac{L_R}{L_c} \right)^3 - \left( \frac{3h_R}{L_c} \right) \right] \]
1st Design Consideration: 

Elastic Limits of Hinge and Column

Compare maximum stresses developed in column and fuse bars

Moment Diagram

Maximum moment in column

Moment about hinge
Comparing Strength of Hinge to Column

Stress in both fuse and column can be expressed as \( f(P) \)

Fuse bar stress

\[
\sigma_{axial} + \sigma_{bending} = \sigma_{\text{max}}
\]

Column Stress

\[
\sigma_{\text{axial}} = \frac{F_{axial}}{A} = \frac{Er \cdot PL}{l_c \cdot K_\theta}
\]

\[
\sigma_{bending} = \frac{M_{bending}}{S} = \frac{EI \cdot PL}{lS \cdot K_\theta}
\]

\[
\sigma_{\text{max}} = \sigma_{\text{axial}} + \sigma_{bending} = f(P)
\]

\[
\sigma_{\text{max}} = \frac{M}{S} = \frac{PL_c}{S_{xx}}
\]
2nd Design Consideration: Determine deflection at top of column

- Stiffnesses of hinge and column act in series

\[ \delta_\theta \equiv \text{defl. due to hinge rotation} \]
\[ \delta_\theta = \theta \times L = \frac{PL^2}{K_\theta} \]

\[ \delta \text{\_flex} \equiv \text{defl. due to flexure of column} \]
\[ \delta \text{\_flex} = \frac{P}{K \text{\_flex}} \]
Assumed small angle changes when relating hinge rotation to axial strain of fuse bars

Compare axial elongation equations:

<table>
<thead>
<tr>
<th>Small Angle Approx.</th>
<th>General Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta = \theta r$</td>
<td>$\Delta = 2 \frac{r}{\cos \beta} \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{\theta}{2} + \beta \right)$</td>
</tr>
</tbody>
</table>

Comparison of Results at Theoretical Elastic Limit of Hinge

<table>
<thead>
<tr>
<th></th>
<th>Small Angle Approx</th>
<th>General Case</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$ (in)</td>
<td>0.0102623</td>
<td>0.0102595</td>
<td>0.03%</td>
</tr>
<tr>
<td>$\delta_{tot}$ (in)</td>
<td>0.76032</td>
<td>0.76063</td>
<td>0.04 %</td>
</tr>
<tr>
<td>$P$ (lb)</td>
<td>1418.85</td>
<td>1418.48</td>
<td>0.03 %</td>
</tr>
</tbody>
</table>
Section III: Experimental Results
Overall Setup:

- Displacement-driven actuator attached to top of column
- Actuator pinned at either end
  - No axial force generated in column
Initial Test Setup:

- **L1 = Load P**
- \( D_1 = \delta_{\text{tot}} \)

\[
\theta = \arctan\left( \frac{D_2 + D_3}{x} \right)
\]

Gauge Location and Interpretation
Experimental Test Pattern

- Experiment ran using a quasi-static cyclic deflection pattern

- Maximum top displacement: 0.185”
- 12 cycles of increasing amplitude
- Total test run time 125 seconds
- Fuse bars were not yielded in test due to limited moment capacity of bottom load cell
Initial Experimental Results

- Stiffness of column-hinge system lower than expected

Theoretical
\[ K = 1866 \, \text{lb/in} \]

Experimental
\[ K = 1055 \, \text{lb/in} \]
Analysis of Initial Results

- Rotational stiffness much less than expected from model
Analysis of Initial Results

- Flexural stiffness matched analytical predictions

<table>
<thead>
<tr>
<th>Comparison of analytical and experimental flexural stiffness</th>
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<tr>
<td></td>
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<tr>
<td>$K_{flex}$ (lb/in)</td>
</tr>
</tbody>
</table>

- Model of column flexure accurate
- Rotation is the source of extra deflection
- Rotational behavior requires more data
2nd Test Setup:

- Potentiometers added between the top and bottom of hinge
- Measured relative motion of top and bottom plates of hinge
- Separates hinge rotation from remainder of base fixture
Analysis of Results: 2nd Test

- Rotation of hinge now comparable to model

<table>
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<tr>
<td></td>
</tr>
<tr>
<td>Experimental</td>
</tr>
<tr>
<td>$K\theta$ (kip/rad)</td>
</tr>
</tbody>
</table>

- Locating source of additional rotation requires more data
3rd Test Setup:

Krypton Coordinate Measurement Machine

- 12 LED’s placed throughout structure to monitor displacement
- Camera system tracks motion of LEDs in 3-D
Analysis of Results: 3rd Test

- Separately tracked motion of each component below hinge
- Located source of extra rotation $\rightarrow$ Load cell at base
Energy Dissipation During Testing

- Hysteresis present in loading/unloading cycles despite absence of fuse-bar yielding
- Possibly caused by some small component yielding or slipping
- Location of component is unknown
  - Large number of interfaces bolted together
Energy Dissipation

- Work done on setup by actuator is equal to area within hysteresis
- Yielding/slipping of component absorbs this energy
- Energy dissipated is the net work done on the column-hinge system
Section IV: Conclusions
Conclusion: Energy Dissipation

- Energy dissipation protects column from excess strain and fatigue
- Fuse-bars designed to be replaced easily after they wear out
- Additional energy can be absorbed by small component yielding or slipping
  - Not desired, may wear out component over time
  - This should be subtracted out when analyzing function of hinge
Conclusion: Design of Plastic Hinge

- Position, axial and bending stiffness, and yielding strength of fuse bars determines moment capacity of hinge.
- Compare to moment capacity of column given a lateral load at the top.
- Effective lengths for different types of stress vary; sensitive to manner of attaching fuse bars.
Conclusion: Deflection at Top of Column

• Calculation of deflection requires:
  – Rotational stiffness of hinge
  – Flexural stiffness of column
  – Sum deflections in series

• Complicated by:
  – Effective length of fuse-bars
    • Determined by method of attaching bars
  – Non-rigid components
    • Load cell
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